

# DIAGONAL PADÉ APPROXIMANT OF THE ONE-BODY GREEN'S FUNCTION

Stefano Di Sabatino

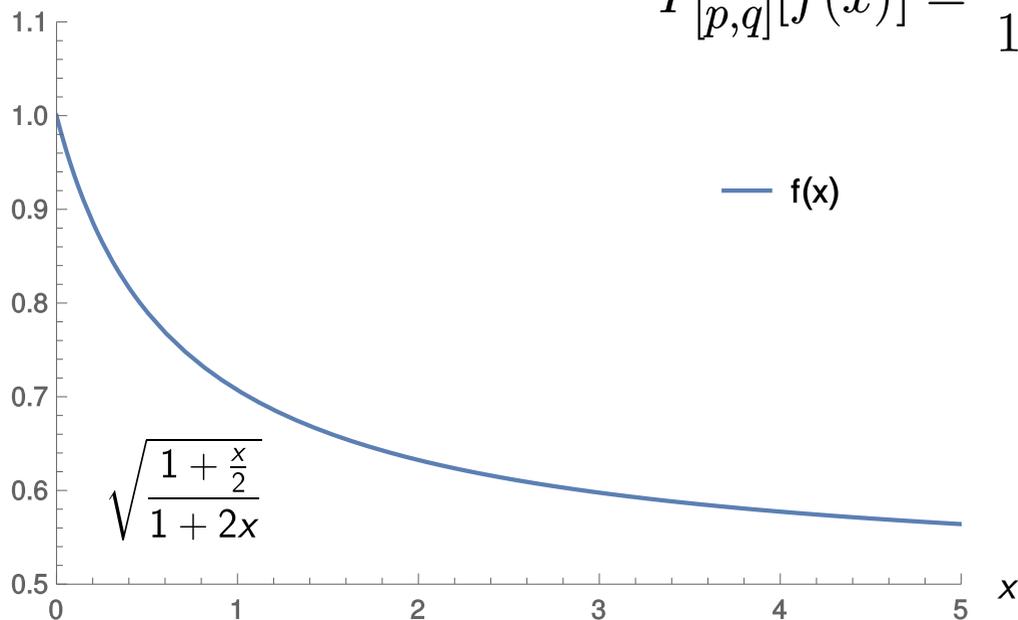
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European Theoretical Spectroscopy Facility



# Padé approximant

$$T_n[f(x)] = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

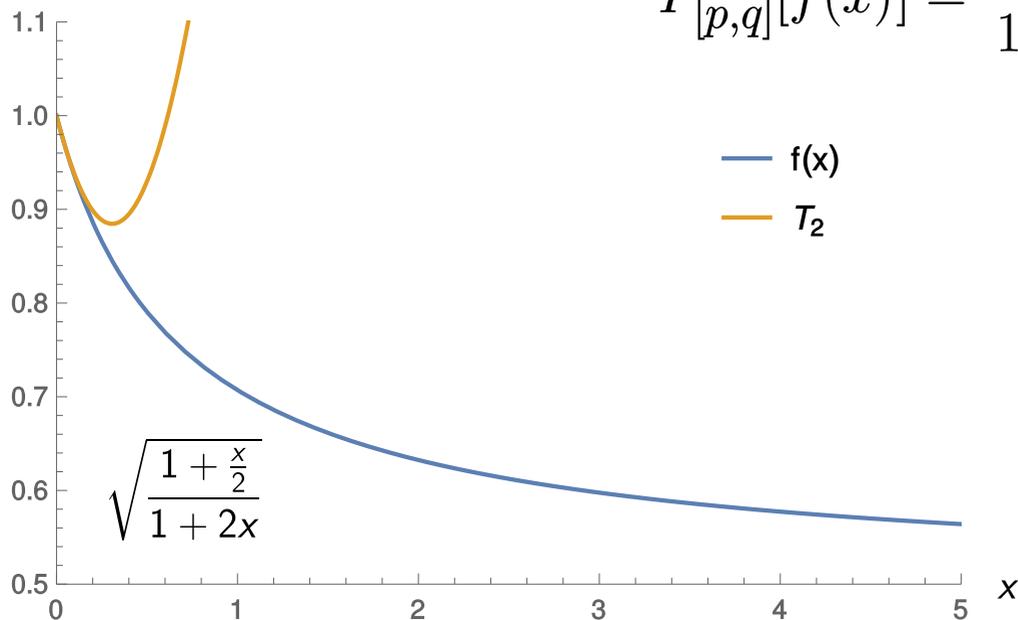
$$P_{[p,q]}[f(x)] = \frac{b_0 + b_1x + \dots + b_px^p}{1 + c_1x + \dots + c_qx^q}$$



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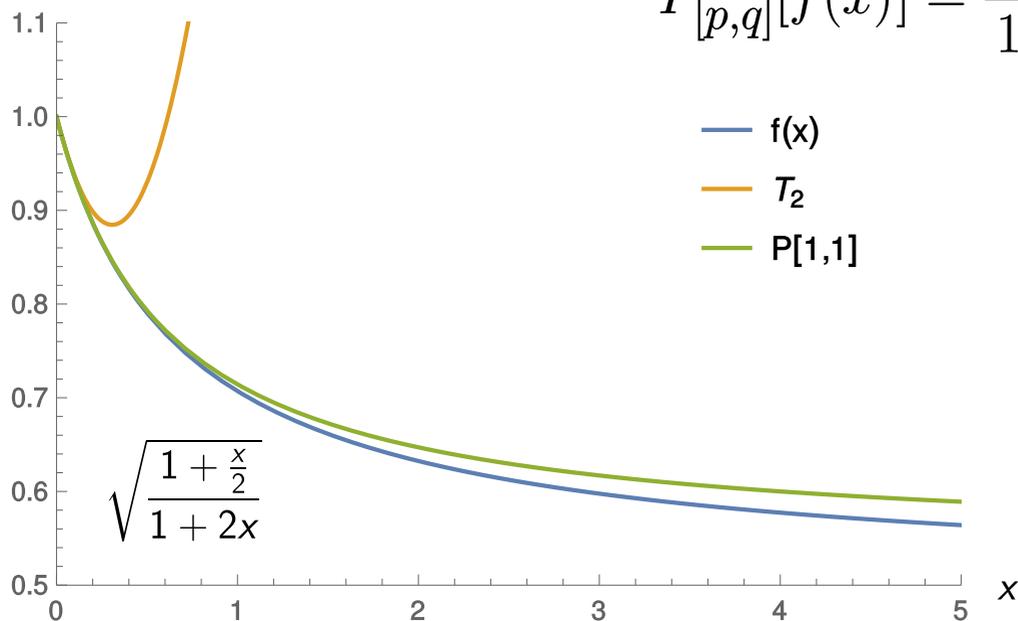
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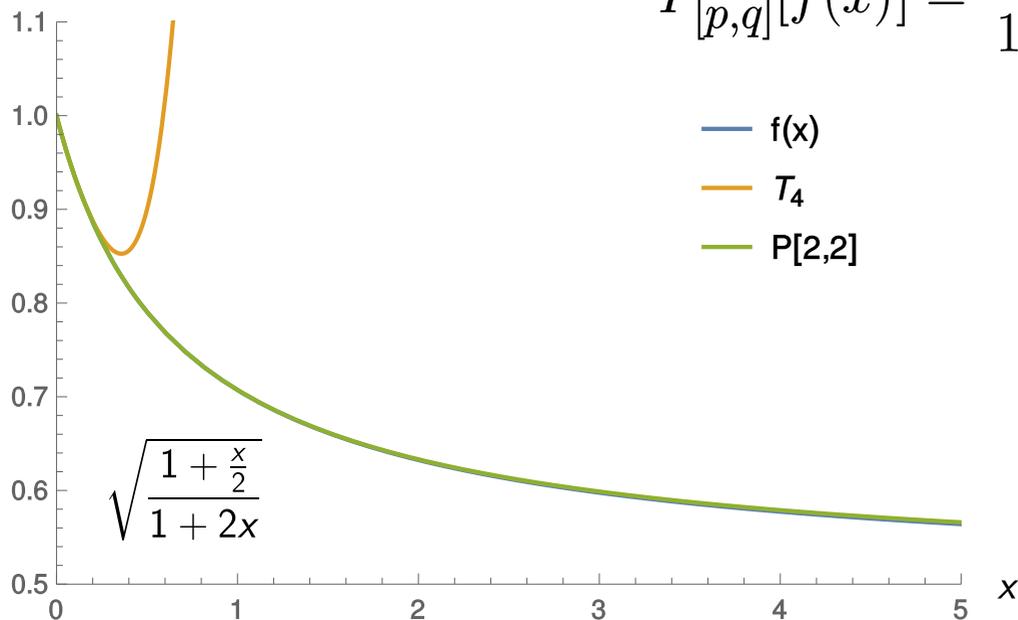
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## Diagonal Padé approximant of the one-body Green's function: A study on Hubbard rings

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Padé approximants to the many-body Green's function can be built by rearranging terms of its perturbative expansion. The hypothesis that the best use of a finite number of terms of such an expansion is given by the subclass of *diagonal* Padé approximants is here tested, and largely confirmed, on a solvable model system, namely the Hubbard ring for a variety of site numbers, fillings, and interaction strengths.

DOI: [10.1103/PhysRevB.99.075149](https://doi.org/10.1103/PhysRevB.99.075149)

# Many Body Perturbation Theory

Perturbation expansion of  $G$

$$G = G_0 + G_0 v G_0 G_0 + G_0 v G_0 G_0 v G_0 G_0 + \dots$$

Dyson equation

$$G = G_0 + G_0 \Sigma G \qquad G = [G_0^{-1} - \Sigma]^{-1}$$

Perturbation expansion of  $\Sigma$

$$\Sigma = v G_0 + v G_0 G_0 v G_0 + \dots$$

# Many Body Perturbation Theory

$$\Sigma = \Sigma^{(1)} + \Sigma^{(2)} + \dots$$

$$\Sigma^{(1)} = \text{[diagram 1]} + \text{[diagram 2]} + \dots$$

$$\Sigma^{(2)} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]} + \text{[diagram 5]} + \text{[diagram 6]} + \text{[diagram 7]} + \text{[diagram 8]}$$

# Diagonal Padé

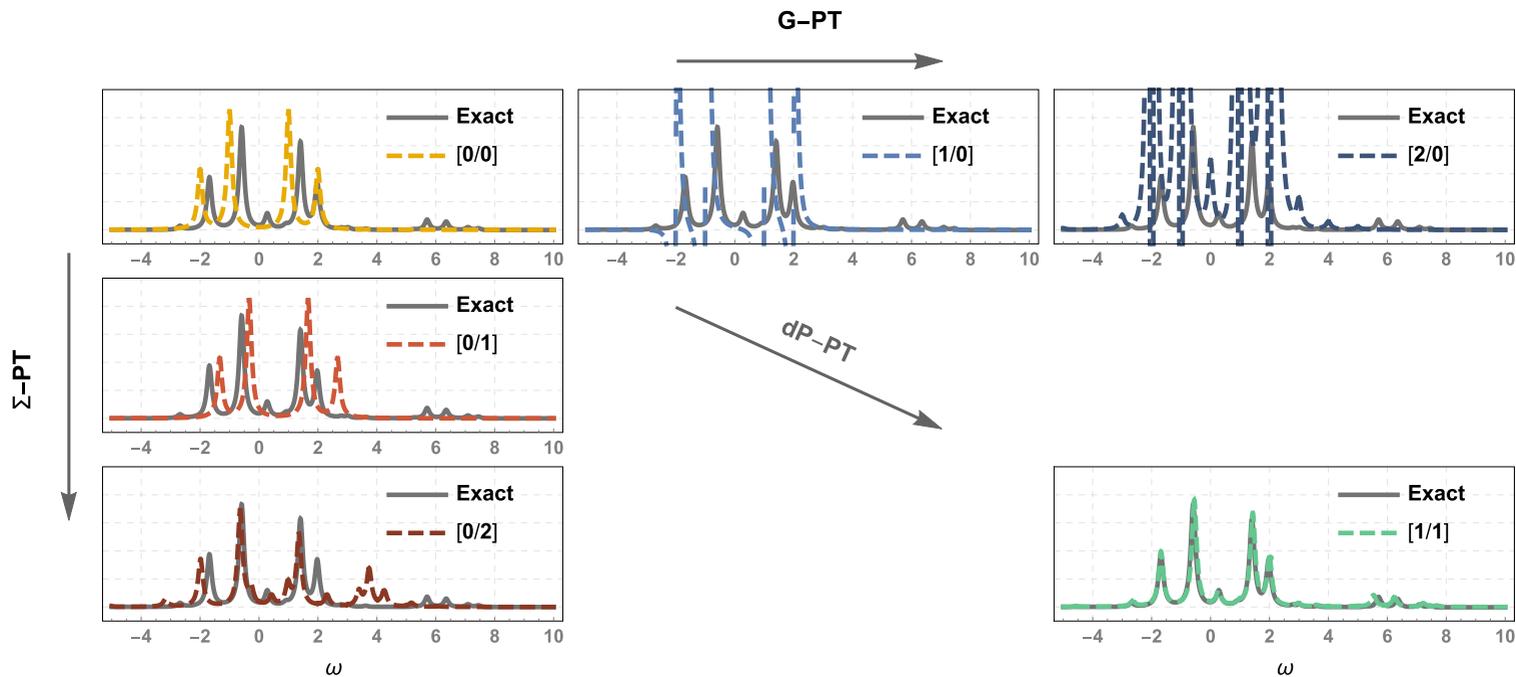
$$\Sigma \approx \Sigma_{[1,1]} = \frac{[\Sigma^{(1)}]^2}{\Sigma^{(1)} - \Sigma^{(2)}}$$

$$\Sigma_{ij}^{(1)} = \sum_{k \in \mathcal{O}} (v_{ikkj} - v_{ikjk}) - i\delta_{ij} v_{ii}^{m.f.}$$

$$\begin{aligned} \Sigma_{ij}^{(2)}(\omega) = & \sum_{o,q,r} \mathcal{O}_q \left( \frac{\mathcal{O}_o \mathcal{U}_r}{\epsilon_o + \epsilon_r} + \frac{\mathcal{O}_r \mathcal{U}_o}{\epsilon_r + \epsilon_o} \right) (v_{iojr} - v_{iorj}) (v_{qroq} - v_{qrqo}) \\ & + \sum_{nps} v_{isnp} (v_{npps} - v_{npsj}) \left( \frac{\mathcal{O}_s \mathcal{U}_n \mathcal{U}_p}{\omega + i\eta - \epsilon_s - \epsilon_n - \epsilon_p} + \frac{\mathcal{O}_n \mathcal{O}_p \mathcal{U}_s}{\omega - i\eta + \epsilon_p + \epsilon_n + \epsilon_s} \right) \\ & + \sum_{n,p} (-i) \left( \frac{\mathcal{O}_n \mathcal{U}_p}{\epsilon_n + \epsilon_p} + \frac{\mathcal{O}_p \mathcal{U}_n}{\epsilon_p + \epsilon_n} \right) (v_{injp} - v_{inpj}) v_{pn}^{m.f.} \end{aligned}$$

# Hubbard model

$L=6, N=2, U/t=4$

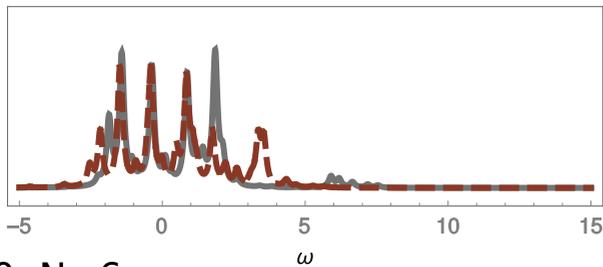


# Hubbard model

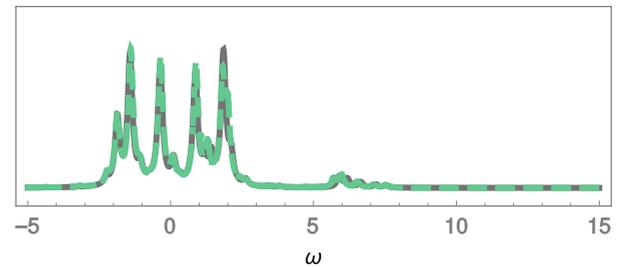
$U=4$

$L=10, N=2$

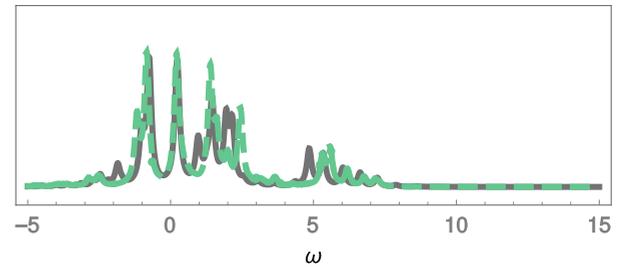
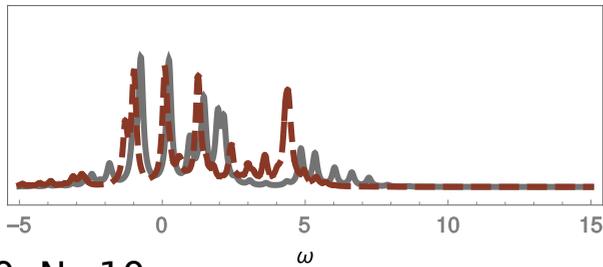
$\Sigma$ -PT [0/2]



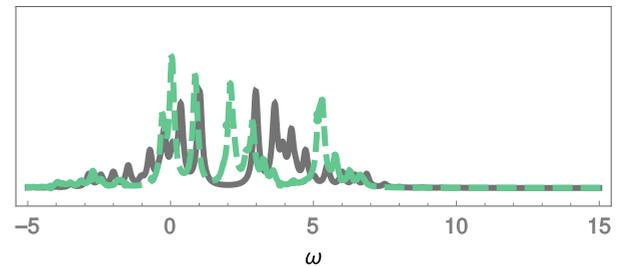
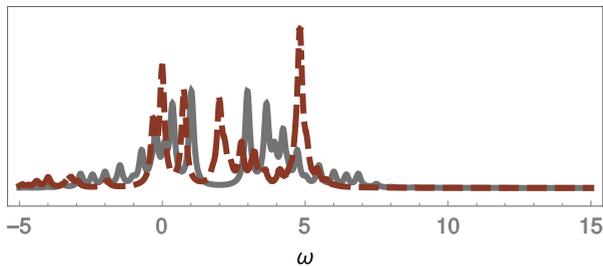
P[1/1]



$L=10, N=6$



$L=10, N=10$

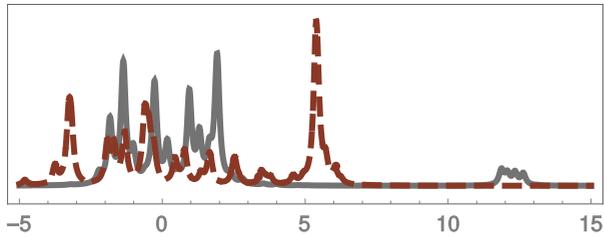


# Hubbard model

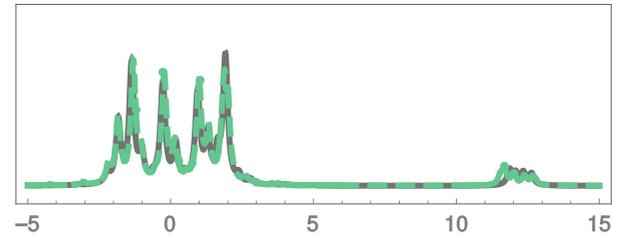
$L=10, N=2, U=10$

$U=10$

$\Sigma$ -PT [0/2]



P[1/1]



# Outlooks

- Padé in realistic systems

Finite or bulk systems?

- Include higher-order diagrams