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Non-Hermitian physics and PT symmetry

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In recent years, notions drawn from non-Hermitian physics and parity–time (PT) symmetry have attracted considerable attention. In particular, the realization that the interplay between gain and loss can lead to entirely new and unexpected features has initiated an intense research effort to explore non-Hermitian systems both theoretically and experimentally. Here we review recent progress in this emerging field, and provide an outlook to future directions and developments.

Conservation of energy is a fundamental concept that shapes our understanding of physical reality. In quantum physics, this requirement demands that a closed system exhibits real eigenenergies—naturally leading to theoretical formulations based on Hermitian Hamiltonians. In many situations, however, one is not interested in the system as a whole, but only in a limited subspace of it. In this case, energy can be exchanged between this specific quantum subsystem and its environment. One of the pioneering studies on the behaviour of such open quantum systems was that of George Gamow on alpha decay[1], where he showed that a particle may escape the nucleus via tunnelling at a rate that can be effectively described through a complex energy eigenvalue. In doing so, he found that the real and imaginary parts of these eigenvalues are related to the experimentally observed energy levels and widths of the corresponding nuclear resonances. In a subsequent work, complex (non-Hermitian) potentials were introduced by Feshbach, Porter and Weisskopf, to model scattering interactions between neutrons and nuclei[2]. In this case, the imaginary part of the potential was responsible for neutron absorption and the formation of compound nuclei.

From microscopic to macroscopic non-Hermiticity

Whereas in these early attempts the use of non-Hermiticity was to a great extent phenomenological, much work was thereafter invested in providing a more formal basis for describing the dynamics of open quantum systems like, for example, the Lindblad formalism[3]. These more rigorous approaches indicate that when a quantum system couples to a surrounding environment or bath, the dynamics of the subsystem itself becomes non-Hermitian and features quantum jumps. In this respect, accounting for the noise induced by these jumps is necessary to keep the quantum mechanical commutation relations intact[4]. Whereas for microscopic systems such a quantum description is crucial, macroscopic settings can be well described by semi-classical approaches in which the noise terms are neglected. At this mean-field level, the process of quantum dissipation can be encapsulated in a non-Hermitian 'effective Hamiltonian'[5–11]. This strategy provides valuable insight into new classes of phenomena that are by nature difficult to address utilizing standard Hermitian representations. In fact, adding a non-Hermitian component to a Hamiltonian does significantly more than merely broadening its resonances and allowing its eigenstates to decay. In particular, it was realized that eigenmodes could merge with each other at so-called 'exceptional points' (also known as branch-point singularities)[7,8,11]. What is remarkable about these degeneracies is that, not only the eigenvalues coalesce at these points (in both their real and imaginary parts), but also their corresponding eigenvectors become completely parallel.

An important milestone in the physics of non-Hermitian systems was the discovery, by Carl Bender and Stefan Boettcher, that a large class of non-conservative Hamiltonians can exhibit entirely real spectra as long as they commute with the parity–time ($\hat{P}\hat{T}$) operator (see Box 1)[12]. To some extent, this counterintuitive result goes against the commonly held view that real eigenvalues are associated only with Hermitian observables. Starting from the aforementioned premise, one can show that a necessary (but not sufficient) condition for PT symmetry to hold is that the complex potential involved should satisfy $V(r) = V^*(-r)$[12], where r is the position vector. In such pseudo-Hermitian configurations the eigenfunctions are no longer orthogonal—that is, $\langle m|n\rangle \neq \delta_{mn}$—and hence the vector space is skewed. Even more intriguing is the possibility of a sharp symmetry-breaking transition once the non-Hermiticity parameter exceeds a certain critical value. In this latter regime, the Hamiltonian and the $\hat{P}\hat{T}$ operator no longer display the same set of eigenfunctions (even though they commute). As a result, the eigenvalues of the system cease to be real. This PT-symmetry-broken phase is associated with the appearance of exceptional points[13].

PT symmetry meets optics

Whereas the implications of these mathematical developments have been a matter of theoretical debate[13–15] for many years, it has been recently recognized that optics can provide a fertile ground where PT symmetric concepts can be fruitfully exploited[16–20]. In optical systems, PT symmetry can be readily established by judiciously incorporating gain and loss—that is, the refractive index profile now plays the role of the complex potential (see Box 1 and Fig. 1). This paradigm shift subsequently led to a flurry of similar research activities in other physical settings, such as electronics[21], microwaves[22] mechanics[23], acoustics[24], and atomic systems[25–27], to mention a few.

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Box 1 | Exceptional points in various settings.

Non-Hermitian phenomena are ubiquitous in nature—they appear even in low-dimensional settings. An analytically tractable example is that of a two-level system describing the physics of two coupled sites with gain and loss. In this case, the Hamiltonian operator is given by:

$$\hat{H} = \begin{pmatrix} \delta - ig_1 & \kappa \\ \kappa & ig_2 \end{pmatrix}, \qquad \hat{H}=|\phi_{1,2}\rangle = E_{1,2}|\phi_{1,2}\rangle \qquad (1)$$

where κ is the coupling constant, δ is a detuning parameter, and g_1, g_2 describe the amount of dissipation/amplification per site. When $g_1 = g_2 = 0$, the eigenvalues are real and the energy is conserved during time evolution. On the other hand, if the complex diagonal terms (g_1, g_2) are non-zero, the matrix is non-Hermitian, and therefore the resulting eigenvalues (E_n) can in general be complex, leading to a time evolution with amplification or dissipation. In this case, the corresponding right eigenvectors are no longer orthogonal. The left eigenvectors can, in turn, be obtained from the adjoint problem $\hat{H}^\dagger|\tilde{\phi}_{1,2}\rangle = E^*_{1,2}|\tilde{\phi}_{1,2}\rangle$. The orthogonality between the right and left modes is expressed by the bi-orthogonality relation ($\langle\tilde{\phi}_{1,2}|\phi_{1,2}\rangle = 0$). An interesting scenario arises at the so-called exceptional point, occurring at $\delta = 0$ and $\kappa = |g_1 + g_2|/2$. At this parameter point, not only are the two eigenvalues identical (see Fig. B1), but also the two eigenvectors become completely parallel.

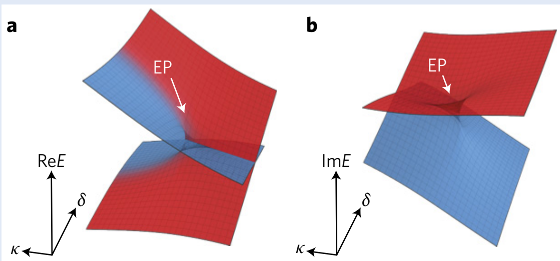

Figure B1 | Riemann surfaces of the complex eigenvalues of the matrix $\hat{H}$ versus the (δ, κ)-parameters. Real (**a**) and imaginary (**b**) part of the eigenvalues.

For the special case of two fully tuned ($\delta = 0$) coupled sites having equal gain and loss ($g_1 = g_2 = g$), the Hamiltonian $\hat{H}$ from equation (1) becomes parity–time (PT) symmetric. This unique class of non-Hermitian Hamiltonians, discovered in 1998, is capable of exhibiting entirely real spectra. More specifically, when $g/\kappa < 1$, the two eigenvalues are real and are given by $E_{1,2} = \pm\kappa\cos\theta$, where $\theta = \sin^{-1}(g/\kappa)$, and the biorthogonal eigenvectors are $|\phi_{1,2}\rangle = [1 \ \ \pm e^{\pm i\theta}]^T$. In this regime, better known as a 'PT exact phase', the two eigenmodes are distributed over both sites and neither of them experiences a net gain or loss—that is, they remain neutral (see Fig. B2). On the other hand, the characteristics of the supermodes drastically change as soon as $g/\kappa > 1$, beyond which the system enters the 'PT-broken phase'. In this regime, the eigenvalues are expressed by $E_{1,2} = \pm i\kappa\sinh(\theta)$, where $\theta = \cosh^{-1}(g/\kappa)$, and their corresponding non-orthogonal eigenvectors turn out to be $|\phi_{1,2}\rangle = [1 \ \ ie^{\pm\theta}]^T$. The symmetry of each mode is broken such that one of them resides mostly in the gain cavity (and hence enjoys amplification) while the other one inhabits the lossy resonator (experiences attenuation), as Fig. B2 graphically depicts.

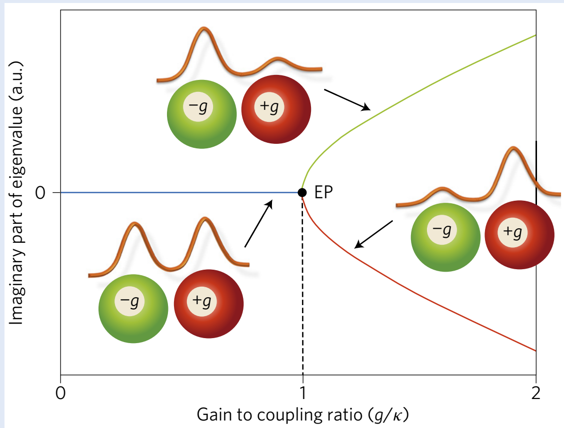

Figure B2 | Plot of the imaginary part of both eigenvalues of $\hat{H}$ versus g/κ when $\delta = 0$.

More generally, a continuous Hamiltonian H is PT symmetric provided that its complex potential satisfies $V(x, y) = V^*(-x, -y)$. An intriguing aspect of such Hamiltonians is the sharp symmetry-breaking transition taking place at an exceptional point. In the broken regime, the Hamiltonian and the PT operator no longer display the same set of eigenfunctions even though they still commute, a direct outcome of the antilinear nature of the time reversal operator. As a result the eigenvalues of the system cease to be real; instead they appear as complex conjugate pairs.
A prototypical system that exhibits the above characteristics is that of a PT-symmetric lattice. An example of a spatial two-dimensional periodic PT potential is given by $V(x, y) = \cos^2 x + \cos^2 y + iV_0 \sin(2x)\sin(2y)$, with the corresponding PT-symmetric Hamiltonian $\hat{H} = -\nabla^2 - V$ (see Fig. B3a).
Here, because of PT symmetry, the band gaps can entirely close and the two first bands can merge, forming a closed surface exhibiting complex conjugate pairs in the overlapping regions and real eigenvalues in the areas in between (see Fig. B3b,c).

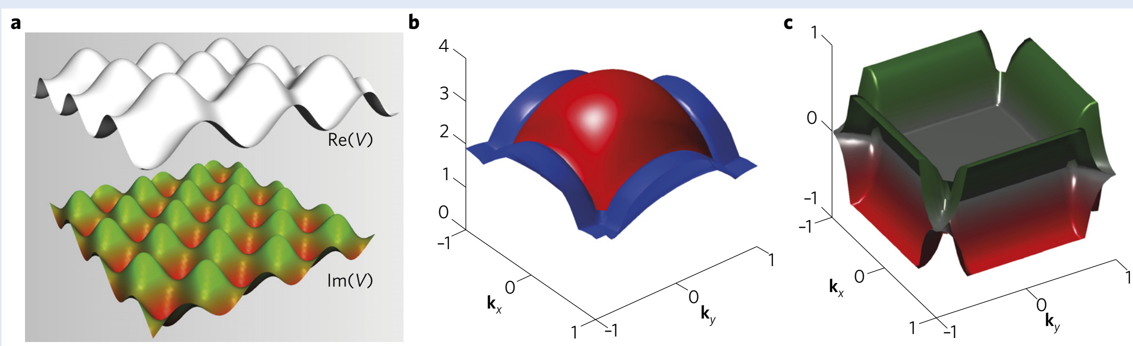

Figure B3 | **a**, PT lattice. **b**,**c**, Real and imaginary parts, respectively, of the first two bands in the broken PT phase.

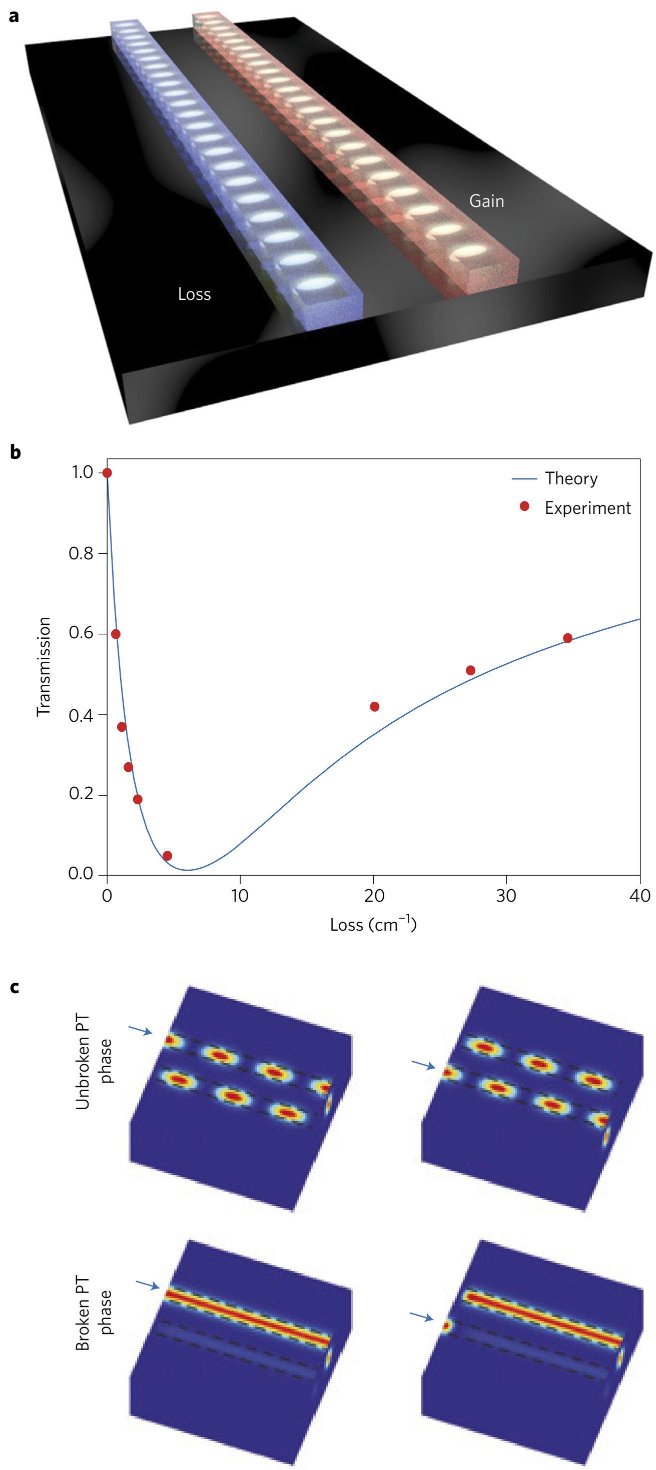

Figure 1 | Schematic presentation of a PT-coupled system and first experimental observations of spontaneous PT symmetry breaking. **a**, A coupled arrangement of two identical waveguide elements having an equal amount of gain and loss. **b**, Loss-induced transparency observed in passive PT-symmetric coupled waveguide channels. In the PT-broken phase, an increase in loss enhances the overall transmission. **c**, Optical wave propagation when the coupled system in **a** is excited at either channel, above and below the PT-symmetry transition threshold. Adapted from ref. 19, APS (**b**); and ref. 20, Macmillan Publishers Ltd (**c**).

To summarize some of the important aspects of non-Hermitian physics and PT symmetry in this review article, we primarily focus on the interplay between conservative and non-conservative processes in photonics. Historically, moulding the flow of light largely relied on our ability to shape the refractive index distribution, an endeavour that started with the first lenses in antiquity and by now has reached such a level of perfection that artificial structures as diverse as optical fibres, photonic crystals, and metamaterials have become possible[28–31]. After the invention of the laser, optical amplification or gain has in turn enabled numerous applications in many areas of science and technology. Gain is unquestionably a valuable commodity in optics, typically used to mitigate losses and to boost the level of a signal. On the other hand, loss, unlike the other two components, is still perceived to be a foe—an undesirable attribute that should be avoided or compensated at all costs. It is in this context that the introduction of PT symmetry (see Box 1) and, in general, non-Hermitian notions in optics, has drastically changed our views on the role of amplification and attenuation in light transport.

Unidirectional invisibility and PT-symmetric metamaterials

The most common methodologies for designing optical metamaterials are those that rely on altering or sometimes even reversing the sign of the constitutive permittivity and permeability parameters ε, μ—something that typically comes at the price of adding undesired losses[31]. In this respect, approaches based on PT symmetry and non-Hermitian physics can introduce additional degrees of freedom in designing metamaterials by allowing one to involve the entire complex plane instead.

One-dimensional periodic structures provide a convenient platform to study the ramifications of PT symmetry on light transport. In their Hermitian embodiment, these arrangements display a sequence of allowed bands and forbidden gaps, where the reflection and transmission remains invariant whether the light enters the structure from the left or right. However, if every unit cell in such a lattice is anti-symmetrically impregnated with gain and loss, the direction that light enters the system can influence its reflection properties. This effect is most pronounced at an exceptional point, where the potential becomes entirely reflectionless from one side. Using a periodic PT grating, one can match this effect with perfect transmission (unity in magnitude, and a phase that is identical to the one in the absence of the grating). In this way, a 'uni-directionally invisible' Bragg grating can be realized, having a response that crucially depends on the way the gain–loss dipoles are oriented (see Fig. 2a). When the direction of the incoming wave or the dipole orientation is flipped, the reflection can even exceed unity[32]. Of course, in linear settings, the transmission coefficients from port to port remain invariant, and hence the reciprocity theorem still holds.

Extensions of these ideas have recently been focused on PT-symmetric gain–loss gratings for 'cloaking' objects[33,34]. The underlying idea is to fabricate gain–loss structures around an object, where the loss part absorbs an incoming wave and the gain part re-emits it. In this respect, a balanced gain–loss distribution can provide relaxed constraints with regard to the size of the concealed object, and enables broadband invisibility. Using such schemes, not only can the scattering amplitudes in the far field be engineered, but also the near-field features can be effectively suppressed through 'constant-intensity waves' that exist even in inhomogeneous scattering landscapes[35,36]—a feature that is inaccessible in Hermitian environments. Negative refraction due to the non-orthogonality of modes and the possibility for aberration-free imaging using PT metasurfaces are relevant topics that could be of significance in nanophotonics[37,38].

Periodic chains of 'PT-atoms' can exhibit a number of other exotic features that have no analogue in Hermitian lattices. These include, for example, band-merging effects and exceptional lines in higher dimensions, as well as phase dislocations and power oscillations[16,39–41]. Other properties of optical PT lattices have been

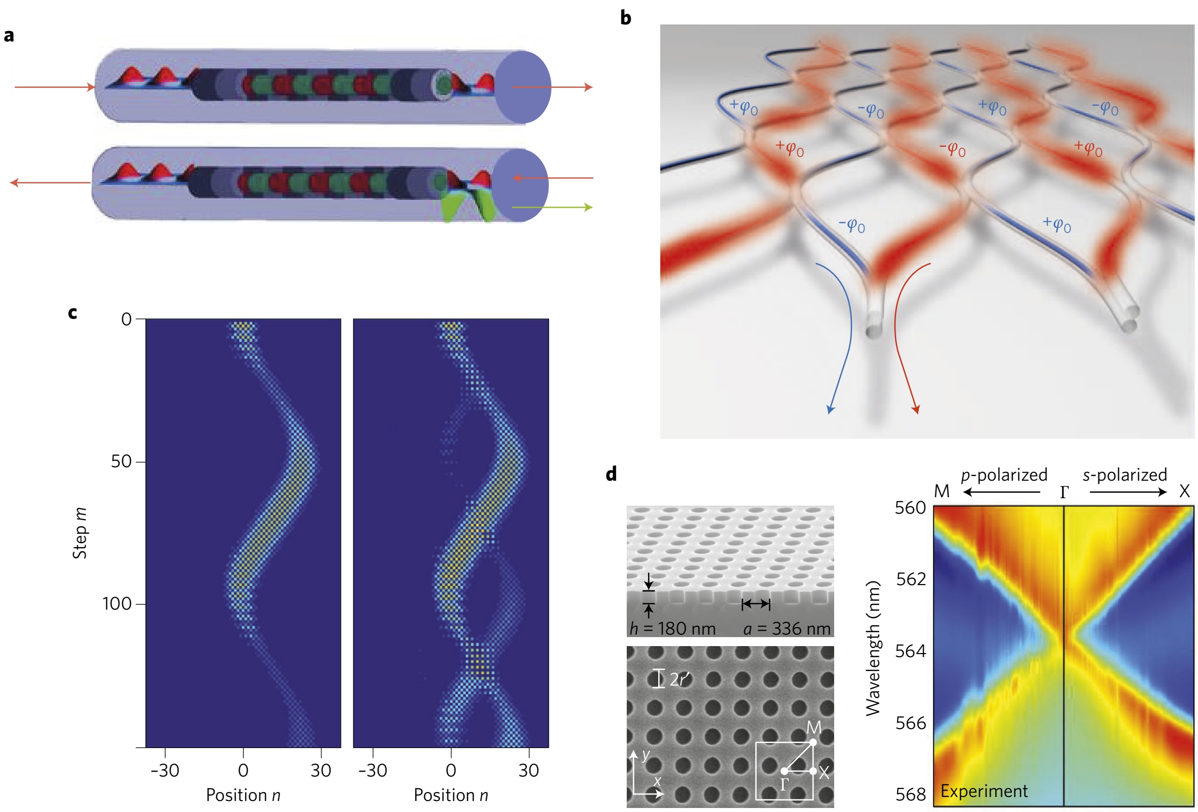

Figure 2 | Unidirectional invisibility and PT-symmetric arrays. a, A PT-symmetric grating can display unidirectional invisibility and a reflectivity exceeding unity. **b**, A gain/loss fibre loop mesh network used to demonstrate unidirectional invisibility. **c**, Bloch oscillations in a passive (left) and PT-symmetric (right) mesh loop system. **d**, A two-dimensional photonic crystal (left) exhibiting rings of exceptional points and the measured reflectivity spectra (right) near such points in the bandstructure. Adapted from ref. 32, APS (**a**); ref. 45, Macmillan Publishers Ltd (**b**,**c**); and ref. 49, Macmillan Publishers Ltd (**d**).

investigated in several theoretical works. These include optical PT graphene arrangements[42], Talbot revivals[43], and Anderson localization[44]. Even though, at first glance, a PT-symmetric lattice with a balanced gain/loss profile may seem to be isotropic, its diffraction patterns resulting from broadband excitation conditions resemble more those expected in anisotropic media. This type of double refraction is a direct outcome of the non-orthogonality of the associated Floquet–Bloch modes[16].

PT synthetic photonic lattices were experimentally realized for the first time in 2012 using coupled fibre loops with balanced gain and loss profiles[45] (Fig. 2b). In this same physical set-up, Bloch oscillations, self-trapped states (Fig. 2c), and defects were also observed, along with double refraction effects, superluminal energy transport, abrupt eigenvalue phase transitions and unidirectional invisibility[45,46]. Similar behaviour was also reported in passive silicon periodic nanowires[47], and subsequently in multilayer Si/SiO_2 unidirectional reflectors[48]. Another relevant development in this area was the demonstration of rings of exceptional points in photonic crystal slabs[49] (Fig. 2d).

Symmetry breaking and exceptional points in laser systems

Lasers provide perhaps the ultimate non-Hermitian playground where gain and loss are simultaneously engaged. In general, gain is induced through external pumping, whereas loss is already present due to the intended out-coupling mechanisms and unavoidable scattering/dissipation in the cavity. The significance of non-Hermiticity in designing and predicting the performance metrics of lasers was long acknowledged by the laser community. For example, an important property associated with a laser device is the emission linewidth. Although to a first-order approximation the laser linewidth can be predicted by the Schawlow–Townes formula[50], in more complex settings, where the eigenmodes are strongly non-orthogonal, a significant departure from this fundamental limit has been predicted[51] and experimentally observed[52]. To a great extent, this linewidth enhancement can be attributed to the loss-induced coupling between modes, which translates into excess noise by the gain mechanism and the cavity feedback[53]. Closely spaced complex eigenvalues were also found to lead to fast self-pulsations in coupled laser oscillators[54], as well as to improved coherent beam combining schemes[55]. Even though some of these aspects have already been partially explored, much insight can still be gained by studying laser systems operating near exceptional points[56].

The peculiar behaviour of complex eigenvalues around an exceptional point manifests itself in the threshold and spectral response of lasing systems[57,58]. When operating a PT-symmetric 'photonic molecule' (Fig. 3a) as a laser, the exceptional points lead to an unexpected pump-induced suppression and revival of lasing[59] - an effect that was experimentally demonstrated in coupled quantum cascade lasers as well as in a pair of silica microcavities with Raman gain[60,61]. Exceptional points were also found to promote single-mode behaviour in another class of oscillators—the so-called dark state lasers[62,63].

In other developments, intrinsically single-mode lasing action was demonstrated in PT-symmetric microcavity arrangements[64–67] (Fig. 3b). Whereas, at a first glance, intentionally introducing additional losses in a resonator may appear to be only detrimental, it is, in fact, common practice to insert lossy apertures in laser cavities to suppress higher-order spatial modes. Once we appreciate the fact that loss can indeed enhance the performance of a laser, the next question naturally is: what is the best strategy to exploit attenuation in a laser cavity to suppress unwanted modes without adversely affecting the fundamental mode? Along these lines, in ref. 65, the

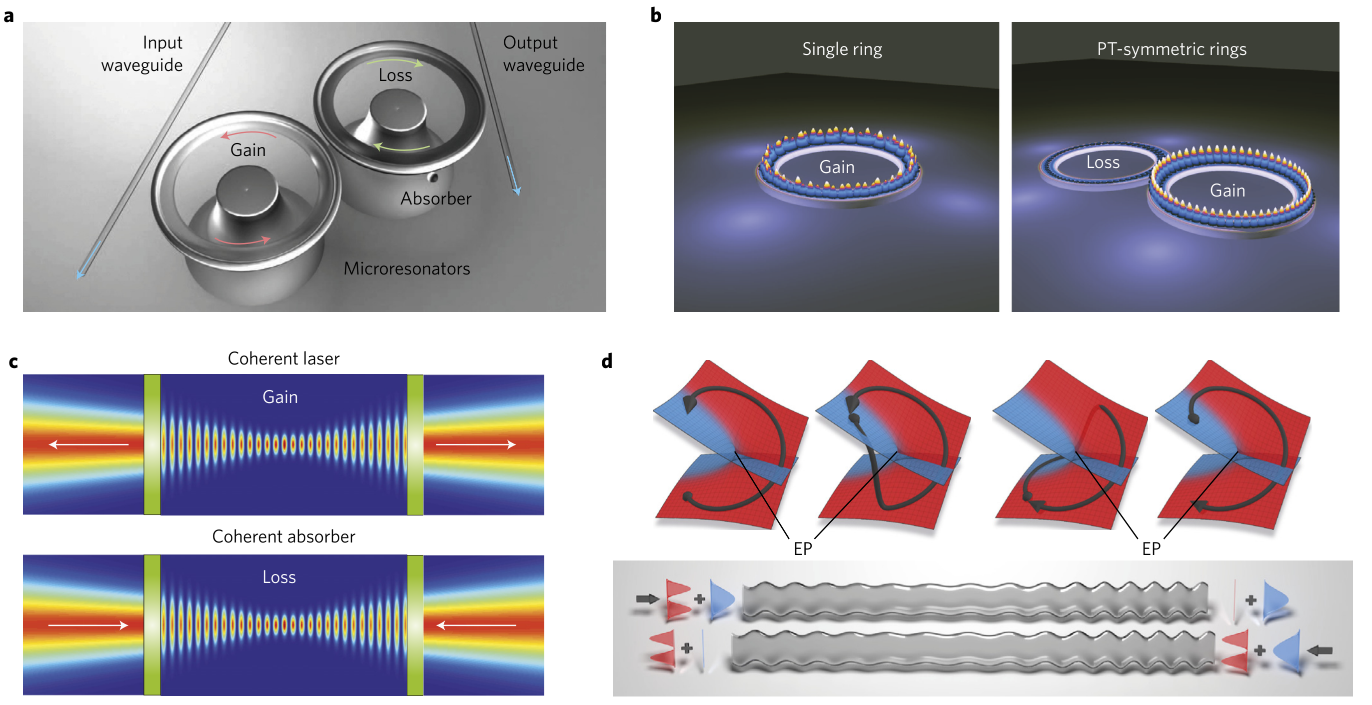

Figure 3 | PT-symmetric lasing systems and exceptional point encirclement. a, Schematic of PT-symmetric coupled resonators. **b**, A PT-symmetric dual microring laser arrangement supporting stable single-mode operation. **c**, The time reverse of a laser with gain (top panel) is a structure with loss (bottom panel) that can perfectly absorb incoming light provided it has the right frequency and phase relationship between the two input beams on the left and right. **d**, Self-intersecting Riemann sheets in complex parameter space with an exceptional point at the centre. Encircling the exceptional point dynamically (curved arrows) faithfully leads to the same final state—depending only on the direction of encirclement. The two plots on the left (right) display the results of a dynamical exceptional point encirclement for a anticlockwise (clockwise) contour. The implementation of this chiral behaviour in a waveguide with a suitably designed boundary and absorber leads to the dominant transmission of a specific mode only (irrespective of the input). Adapted from ref. 65, AAAS (**b**); ref. 73, APS (**c**); and ref. 96, Macmillan Publishers Ltd (**d**).

selective breaking of PT symmetry in a coupled microresonator system was exploited to realize a new family of inherently single-mode lasers with unique self-adapting properties (Fig. 3b). In these micro-scale arrangements, mode selection is accomplished without incorporating dispersive elements as previously done for example in vertical cavity surface emitting lasers (VCSELs) and distributed feedback (DFB) lasers. Without any penalty in terms of slope efficiency or threshold pump intensity, such coupled PT-symmetric lasers can become a viable source of coherent radiation in integrated photonic circuits where single-growth processing and in-plane output coupling are preferred. In a parallel development, thresholdless symmetry breaking was observed in a ring structure involving a PT grating, which in turn led to stable single-mode operation for the desired whispering-gallery mode order[66].

There are several possible future developments. One would be to consider the properties of multiple and individually tunable coupled cavities. Such configurations can exhibit higher-order exceptional points[68] that may in turn be useful in developing high-performance active sensors[69–71]. Exceptional points also play an increasingly prominent role in generating beams with orbital angular momenta[72]. Another possibility is the realization of a PT-symmetric laser-absorber[73,74] that can simultaneously lase and perfectly absorb light at the same frequency (Fig. 3c). Related concepts, such as phonon lasing, are meanwhile being considered in conjunction with PT symmetry[75].

Non-Hermitian nonlinear systems

The interplay between non-Hermiticity and nonlinear processes poses a number of intriguing and fundamental questions. For example, one of the key features of PT-symmetric systems is the abrupt transition of their corresponding eigenvalue spectra from the real to the complex domain. In view of the fact that light by itself can locally modify its index surroundings via nonlinearities, it is reasonable to ask whether the rules governing this behaviour can change at higher power levels. Indeed, it was theoretically predicted that nonlinear Kerr effects can induce a transition in a PT-symmetric lattice from a broken to an unbroken phase, and vice versa[76]. In other words, in sharp contrast with linear PT-symmetric configurations, nonlinear processes are capable of reversing the order in which the symmetry breaking occurs. This feature was successfully observed in semiconductor-based PT-symmetric dual microring laser resonators as a result of gain saturation[77,78]. Yet, even in the nonlinear regime, the resulting non-Hermitian states were found to retain the structural form of the corresponding linear eigenvectors. Such nonlinear PT-symmetric systems can have far-reaching implications. For example, in a recent study, it has been demonstrated that PT-symmetric circuits incorporating nonlinear gain saturation elements can provide robust wireless power transfer[79].

Nonlinear optical transport in periodic lattices endowed with PT symmetry is another topic of interest[18,80]. In this regard, coherent structures in the form of discrete or lattice solitons were identified in quasi-transparent PT periodic optical potentials[81–83]. Contrary to Ginzburg–Landau dissipative systems, where self-trapped waves are known to exist as points in phase space, here, entire families of solutions can emerge as if the PT photonic crystal was conservative[83]. This unusual class of optical solitons was recently observed for the first time in nonlinear mesh lattices, with global and local PT symmetry[84]. Modulation instability in nonlinear non-Hermitian inhomogeneous media was also investigated by employing generalized plane wave solutions[35]. The possibility of simultaneously engaging non-Hermiticity and nonlinearity as a means to implement all-dielectric unidirectional devices was also suggested[85,86].

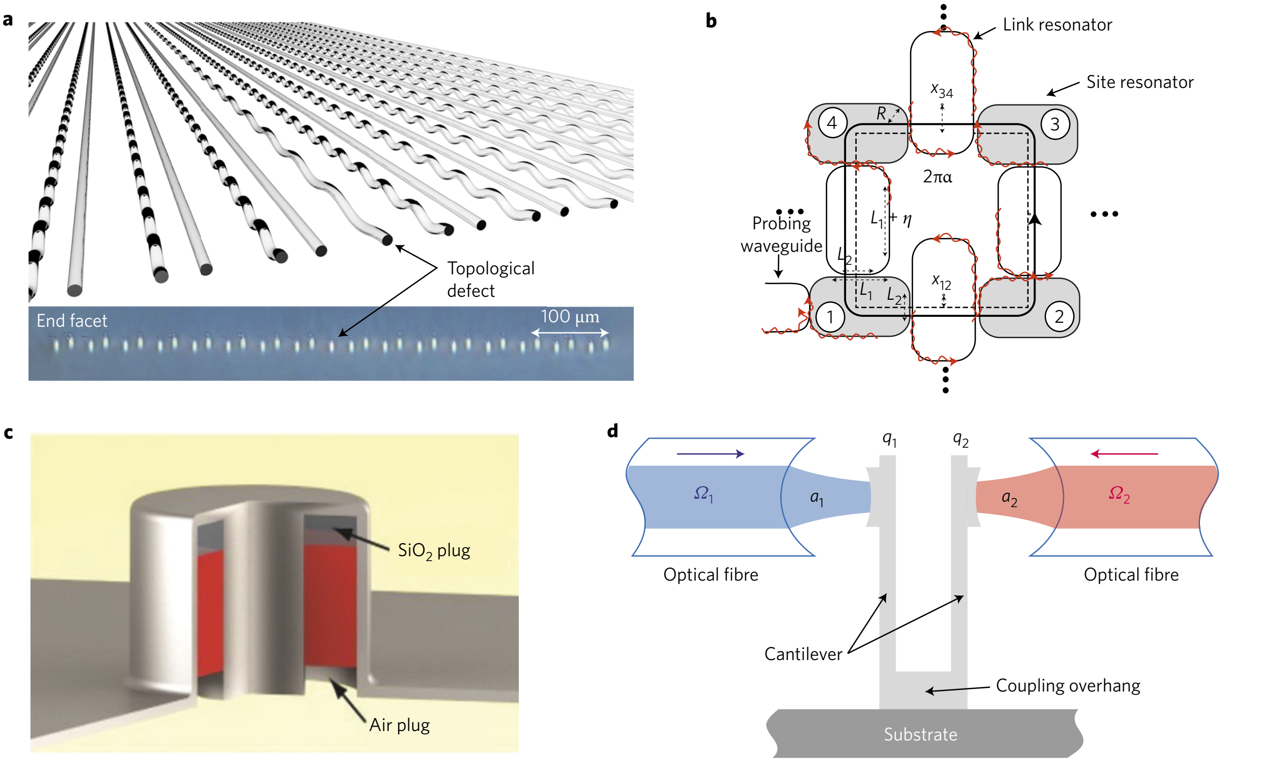

Figure 4 | Future directions in PT-symmetric optics. a, A photonic topological PT-symmetric network using laser-written waveguide structures. **b**, Aperiodic ring resonator topological insulator array where the spatial shift of the link resonators establishes a synthetic magnetic field. **c**, A plasmonic nanolaser based on a subwavelength coaxial design where gain overcomes metal losses. **d**, A possible implementation of a coupled optomechanical system for realizing mechanical PT symmetry. Adapted from ref. 105, Macmillan Publishers Ltd (**a**); ref. 101, Macmillan Publishers Ltd (**b**); ref. 110, Macmillan Publishers Ltd (**c**); and ref. 115, APS (**d**).

Non-Hermiticity can also provide an alternative route to phase matching in three- and four-wave mixing processes[87,88]. This, in turn, opens new opportunities for utilizing semiconductor materials with large nonlinear coefficients for implementing on-chip optical components such as parametric amplifiers and oscillators. Additionally, non-Hermitian systems exhibiting nonlinearities due to the coupling between light and mechanical vibrations have also been recently explored[74,89].

Encircling exceptional points

A fascinating aspect of exceptional points is related to the geometry of the self-intersecting Riemann sheets on which the complex eigenvalues unfold (see Fig. 3d and Box 1). This led to several studies as to what ensues when an exceptional point is encircled along a closed loop in parameter space. Theoretical predictions pointed out that in the quasi-static regime, the system does not return to its initial state upon encirclement, but to a different state on another Riemann sheet—requiring a second encirclement to return to the initial state (apart from a geometric Berry phase of π (ref. 90)). Initial experiments in microwave arrangements[91], optical cavities[92], and polariton condensates[93] confirmed this interesting behaviour. Yet, subsequent studies showed that the gain and loss needed to produce an exceptional point also destroy the adiabatic evolution around the exceptional point[94,95]. In a fully dynamical picture, additional non-adiabatic transitions emerge, which give rise to a fascinating chiral behaviour through which the direction of encirclement alone determines the final state of the system (see trajectories in Fig. 3d).

These latter predictions were confirmed in two experiments, in the microwave domain and in optomechanics[96,97]. In the first experiment[96] (see Fig. 3d), the dynamical encirclement of an exceptional point was mapped onto the spatial propagation in a two-mode waveguide featuring suitably engineered boundary modulations and internal losses. Since the polarity of the exceptional point encirclement is dictated by the propagation direction, the chiral behaviour manifests itself in unique transmission characteristics. Irrespective of the input state, the waveguide transmits predominantly into one mode depending on the direction of propagation. In the second experiment[97], a mechanical membrane was placed inside an optical resonator and an external laser was used to implement the above chiral state transfer. Finally, the possibility of an integrated optical omni-polarizer has been suggested. This component can faithfully produce a specific polarization state regardless of the input—something that is impossible using single channel schemes[98].

Outlook and future directions

Topological phenomena in non-Hermitian optical systems. The past decade has seen a surge of interest in topological insulators: materials that allow extremely robust and uninterrupted flow of current, akin to quantum Hall phenomena. This naturally raised the exciting possibility of investigating topological systems in photonics, where the physics can be studied in great detail and under controlled conditions. Lately, photonic systems have allowed these topological phenomena to be probed directly in ways that would have been inconceivable in the solid state[99–105] (see Fig. 4a,b for two examples). Given that optics offers an ideal ground where gain and/or loss can be physically implemented, a promising future direction is to explore light dynamics in non-Hermitian topological systems. For example, an open question in this field pertains to the existence of topological insulators in PT-symmetric non-Hermitian arrangements[106]. The connection between topological physics and non-Hermiticity has just recently come to be appreciated. In particular, the insight provided by the Rudner–Levitov model, demonstrating that topological phase transitions can take place in one-dimensional non-Hermitian quantum walks[107], paves the way for the experimental realization of topologically protected edge states in optical PT-symmetric lattices, and for examining possible relations between topological Chern numbers and exceptional points. Another important question, still to be addressed, is how chiral aspects in encircling an exceptional point (see previous section) could be used for topological state protection. These intriguing topics could serve as a bridge between the fields of topological photonics and PT-symmetric optics.

A PT-symmetric perspective in active plasmonics. The field of plasmonics provides a route towards confining and manipulating light at the nanoscale. This type of sub-diffraction localization is enabled by electromagnetic excitations coupled to electron charge density waves on a metal–dielectric interface, better known as surface plasmon polaritons. Unfortunately, this energy recycling between photons and charge oscillations is subject to appreciable metal dissipation losses. Research efforts in the field of active nanoplasmonics are meant to address this challenge by introducing gain materials in the vicinity of metals[108–110] (see Fig. 4c for an example). Given that active plasmonics and PT optics share common ground, by merging ideas from these two areas one can perhaps devise methodologies to effectively scale down light generation, transport, and detection into subwavelength dimensions using both metals and gain media.

Several theoretical studies have so far examined such interesting possibilities by considering PT surface waves, PT long-range plasmon polaritons, exceptional points in plasmonic waveguides, as well as PT-symmetry breaking at a metallic interface[111]. PT plasmonics brings up many interesting and fundamental issues. For example, at the subwavelength domain, amplification and attenuation need to be revisited from a quantum perspective. In addition, at this point it is not clear how the local density of states and the Purcell factor will be modified when a PT plasmonic phase transition occurs.

Non-Hermiticity in the quantum regime

Thus far, the application of PT symmetry to optical systems has largely relied on effective medium theories, where processes such as stochastic quantum jumps can be neglected (see introduction). Naturally, a new direction could be the study of non-Hermitian effects in small-scale devices as well as in atomic and molecular systems, where quantum processes are known to play a significant role. These include, for instance, driven atomic condensates in cavities[112], artificial atoms or hybrid quantum systems in cavity quantum electrodynamics[113,114] as well as coupled optomechanical resonators with gain and loss, where effects such as phonon lasing[115] near exceptional points could be explored (Fig. 4d). Initial theoretical studies in this direction show that the presence of quantum noise leads to significantly different physics as compared to that expected from semiclassical approaches. Novel phases with preserved or 'weakly' broken PT symmetry appear, as well as unconventional transitions from high-noise thermal emission to a coherent lasing state[116]. Such interactions could in principle also appear in other contexts of quantum optics, such as the prototypical case of an atom interacting with the mode of a light field in an open system. The widely studied crossover between weak and strong light–matter interaction is mediated through an exceptional point[113]. Indeed, results along these lines have been reported in systems having a single quantum dot embedded in a semiconductor microcavity[114]. Similar dynamics also takes place during radiofrequency transitions between metastable states in atomic hydrogen[117]. Of interest will be to theoretically identify a broader class of quantum dynamical scenarios where transitions can be engineered via controlled dephasing interactions.

Other platforms for non-Hermitian physics

As pointed out in the introduction of this review, by now non-Hermitian concepts have penetrated a number of sub-disciplines in physics, from nuclear and quantum, to optical, microwave, electronic and mechanical systems. One rapidly expanding area is that of acoustics, where exceptional points can be readily accessed[118,119] and used for the realization of invisible sensors[24]. In the quantum realm, PT symmetry can be observed in a number of platforms. These include, for example, Bose–Einstein condensates[112], ferromagnetic superfluids[120], exciton-polariton condensates[93], and flying atoms[27]. Most recently, the discovery of a hidden PT symmetry in the quantum amplification by superradiant emission of radiation (QASER) was reported which employs a parametric resonance to achieve an exponential generation of X-rays[121]. Whereas a variety of different topics have already been reinvigorated within the framework of non-Hermitian physics, the subject is still in its infancy, with many of its possibilities as yet undiscovered.

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