MATHEMATICAL PHYSICS

Circling exceptional points

Going around an exceptional point in a full circle can be a non-adiabatic, asymmetric process. This surprising prediction is now confirmed by two separate experiments.

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Two papers recently published in *Nature* deal with exceptional points[1,2]. Whereas this concept may not be familiar to all physicists, eigenvalues or eigenmodes certainly are. The former comes up in quantum mechanics, whereas the latter is usually used with the same meaning in classical systems. Think of a string of a violin or piano. If struck it produces a specific tone: the vibrational eigenfrequency characterized by the length, the tension and further mechanical properties of the string. The string does not vibrate forever as there is damping. A simple way of describing such a wave pattern is given by the differential equation solutions of the form $\exp(-\lambda t \pm i\omega t)$, where λ is the friction coefficient and $\omega = (f^2 - \lambda^2)^{1/2}$ with f being the frequency without damping and ω the actual frequency modified by the damping.

The eigenmodes of the underlying differential equation are actually the complex numbers $i\lambda \pm (f^2 - \lambda^2)^{1/2}$. The real (or imaginary) part of the solution describes a damped oscillation as long as $\lambda < |f|$. However, if the damping is too strong, that is if $\lambda > |f|$, there is no oscillation. A critical point occurs at $\lambda = |f|$ where we encounter the simplest form of an exceptional point: two independent eigensolutions coalesce. Coalescence is different from the well-known degeneracy in that not only the eigenvalues are equal, but even the eigenfunctions become aligned — linearly dependent. The parameter λ plays a crucial role: the eigenmodes are complex because λ is nonzero. For a given f the frequency ω has a square root singularity in the parameter λ. This simple example illustrates two basic properties of an exceptional point: two eigenvalues coalesce at some parameter value and the eigenvalues have a square root singularity at this critical point.

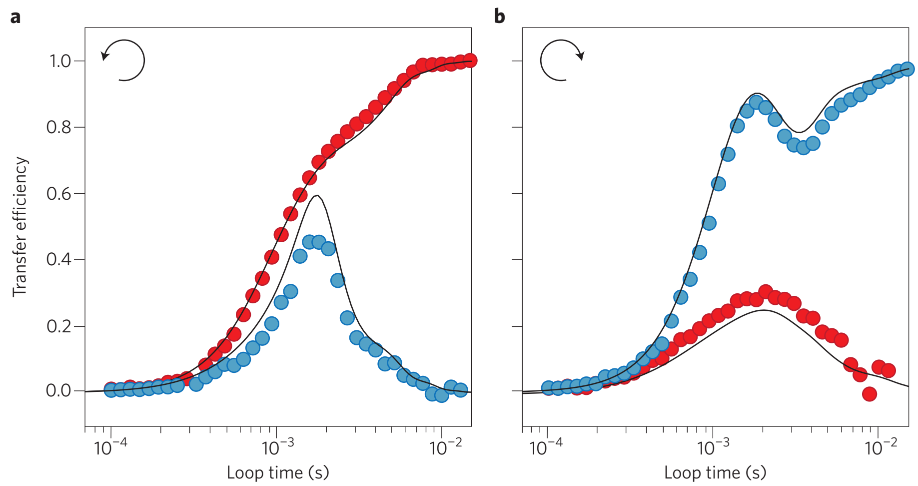

Figure 2 | The transfer efficiency as a function of loop time. **a**,**b**, The measured transfer efficiency of the two different states (labelled red and blue) ends at different values depending on the direction the exceptional point is encircled. For the anticlockwise direction it is the red state (**a**) and for the clockwise direction it is the blue state (**b**) that is driven and attains maximal efficiency after sufficient loop time while the efficiency of the partner state vanishes. The solid lines represent numerical results. Reproduced from ref. 2, NPG.

The two different effects obtained by encircling exceptional points — adiabatic versus non-adiabatic — are best illustrated graphically by two Riemann sheets connected at the square root branch point (Fig. 1). In an actual experiment the complex parameter is replaced by two real parameters and the two sheets represent two eigenvalues with complex values. A suitable parameter change results in encircling the exceptional point. Starting on the upper sheet one ends up on the lower sheet after one round (the line where the sheets penetrate each other is not a discontinuity: the values move smoothly from one sheet to the other). Encircling twice brings one back to the initial point. This smooth switch with no sudden jump is called adiabatic transport, as illustrated on the left-hand side in Fig. 1a and on the right-hand side in Fig. 1b.

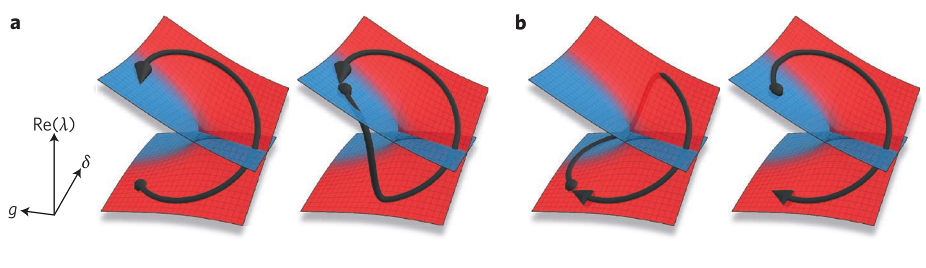

Figure 1 | Dynamically encircling an exceptional point is a non-adiabatic process. **a**,**b**, Anticlockwise (**a**) and clockwise (**b**) trajectories can start on the red (gain) or blue (loss) Riemann sheets for the two states. The end points depend only on the direction of the encircling and not on the starting point. A non-adiabatic jump is clearly discernible on the right-hand side of **a** and on the left-hand side of **b**. The state is attracted by and jumps to the red sheet. g and δ are the coupling and detuning parameters, respectively. Reproduced from ref. 1, NPG.

It has been argued theoretically[3] that for systems driven by loss and gain there is necessarily a jump: a sudden non-adiabatic switch depending on the direction and on the initial values of the trajectory. This implies an asymmetry between clockwise and anticlockwise transport around an exceptional point (Fig. 2). Or, in other words, the non-adiabatic terms lead to chiral behaviour. Jörg Doppler and colleagues confirm this using two coupled waveguides[1]. And Haitan Xu and co-workers report the same finding in an optomechanical system[2].

Such experiments are extremely challenging. Firstly, as the system is open it interacts with the environment. There is loss (by absorptive material or radiation) and gain (by laser pumping), which must be delicately balanced. Only then will the position of the exceptional point remain stationary in the parameter space. And most importantly, only then will non-adiabatic effects appear (Fig. 1). Secondly, the encircling must be done dynamically, thus requiring a continuous time-dependent change of the parameters. Moreover, the speed of the encircling must also be controlled. This is actually implemented by imposing an appropriate slow change of the boundary parameters along the propagation direction.

The reports of Doppler *et al.* and Xu *et al.* confirm the non-adiabatic and asymmetric nature of encircling exceptional points. These experimental approaches are not limited to electromagnetic waves, but are also applicable to acoustic and other matter waves. The results could be used for quantum control and switching protocols and further studies could explore the behaviour of thermal and quantum fluctuations in the vicinity of an exceptional point, thus opening intriguing directions for further research. ❐

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Published online: 8 August 2016